A Trial Modeling of Perturbed Repeating Earthquakes Combined by Mathematical Statics, Numerical Modeling and Seismological Observations

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1 Introduction: A Review of Previous Studies on Renewal Point Processes

Earthquake recurrence cycle has been thought as repetition of stress accumulation for long term as interseismic stage and stress release for short period as coseismic stage. Without any stress perturbation under constant stress loading rate, earthquakes are expected to become perfectly characteristic in that the recurrence intervals and magnitudes (or moment release amounts) are always constant.

Focusing on the previous record of repeating earthquake activity off Kamaishi, Japan, Matsuzawa et al. (2002) succeeded in forecasting the occurrence time of next repeating earthquake on the ground that recurrence interval was so stable (Igarashi et al. 2003) as to be treated as the normal distribution. This assumption is a good approximation in case of weak stress perturbation condition due to isolated asperities far from large earthquake source regions (Uchida et al. 2005). However, the assumption of the normal distribution is just for simplification, which is lack of corroborative scientific evidence.

From these background, the Headquarters for Earthquake Research Promotion (2013) belonging to the Ministry of Education, Culture, Sports, Science and Technology (MEXT) of the Japanese government proposed a physical model for the earthquake recurrence by combining the Brownian vibrational relaxation process

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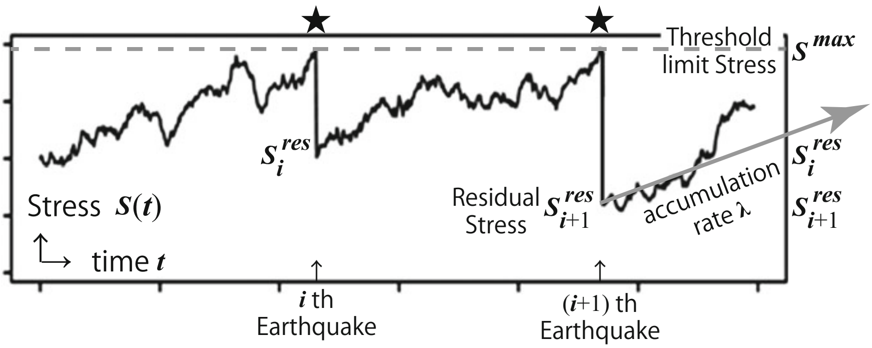

Fig. 1 Schematic illustration of the Brownian Passage Time (BPT) renewal point process, where S^{max} and S^{res} is the threshold limit of loading shear stress and residual shear stress after the occurrence of earthquakes, respectively (modified from Nomura 2015)

and constant threshold limit of stress S^{max} (Fig. 1). This model is described as a Brownian Passage Time (BPT) renewal point process, and stress $S(t)$ at the time t is described as

$$dS(t) = \lambda dt + \sigma dW(t), \tag{1}$$

where the stress accumulation rate (λ) is assumed to be constant value derived from the average of stress release amount per recurrence interval for each event, σ is a perturbation rate to be justified as a dispersion, and $W(t)$ is a standard Brownian motion described as stress perturbation mainly derived from nearby earthquakes.

Equation (1) is developed from the Time Predictable Model (Shimazaki and Nakata 1980), which enables us to evaluate the probability of recurrence time of the next earthquake (x) as follows

$$f(x|\mu, \alpha) = \sqrt{\frac{\mu}{2\pi\alpha^2 x^3}} \exp\left\{-\frac{(x-\mu)^2}{2\mu\alpha^2 x}\right\} (x > 0), \tag{2}$$

where

$$\mu = \left(S^{\max} - S^{\mathrm{res}}_{\mathrm{ave}}\right) / \lambda, \tag{3}$$

$$\alpha^2 = \sigma^2 / \left\{\lambda \left(S^{\max} - S^{\mathrm{res}}_{\mathrm{ave}}\right)\right\}. \tag{4}$$

$$S^{res}_{ave} = \sum_i^n S_i^{res} \Big/ n. \tag{5}$$

2 Application to Perturbed Repeating Earthquakes on the Basis of Trend Renewal Model

The physical model in Fig. 1 means that actual stress rate is described as the combination of the accumulation rate (λ) and the Brownian relaxation process, which is true for the case of small stress perturbation. With significantly great stress perturbation, however, this assumption is not good approximation. For example, repeating earthquake recurrence interval off Kamaishi was temporarily shorter just after the 2011 Mw 9.0 Tohoku earthquake (Fig. 2). This is because the accumulation rate became significantly higher due to large postseismic slip (Ariyoshi et al. 2015).

To consider the acceleration effect of loading rate due to large postseismic slip, Nomura et al. (2014) has improved the Brownian vibrational relaxation process by describing the accumulation rate λ as a function of time $\lambda(t)$ as follows:

$$dS\left(t\right) = \lambda_0 \lambda\left(t\right) dt + \sigma_0 \sqrt{\lambda\left(t\right)} dW\left(t\right), \tag{6}$$

where the non-stationary accumulation rate and dispersion is converted to $\lambda_0 \lambda\left(t\right)$ and $\sigma_0 \sqrt{\lambda\left(t\right)}$, respectively. Equation (6) does not allow Brownian Passage Time (BPT) renewal point process, but it obey BPT distribution by converting time t' as

$$t' = \int_0^t \lambda\left(s\right) ds. \tag{7}$$

The time-transformed renewal process defined by Eqs. (6) and (7) is called as the Trend renewal process (Lindqvist et al. 2003). Nomura et al. (2014) applied the law of Omori-Utsu formula $\lambda\left(t\right) = K\left(t + c\right)^{-p}$ to Eq. (7) by fitting the constant parameters p and c on the basis of aftershock seismicity as shown in Fig. 3.

3 Observation and Modeling of Perturbed Seismic Moment Release for the Source Region of Repeaters

Figure 2 suggests that magnitude of repeating earthquakes off Kamaishi is largely characteristic before the occurrence of the 2011 M9 Tohoku earthquake. Just after the occurrence, however, the magnitude is significantly changed to be not only greater (Fig. 4) but also smaller (as shown in the right side of Fig. 2). This phenomenon means that we should treat the moment release amount as non-stationary. In other words, not only the stress accumulation rate described in the Sect. 2 but also the difference of $S^{max} - S^{res}$ should be not constant.

From recent observation results, Uchida et al. (2015) reported that the repeating earthquakes (M2.5–6.1) show postseismic magnitude increases for most sequences located in the area of large postseismic slip at the downdip extension of the M9

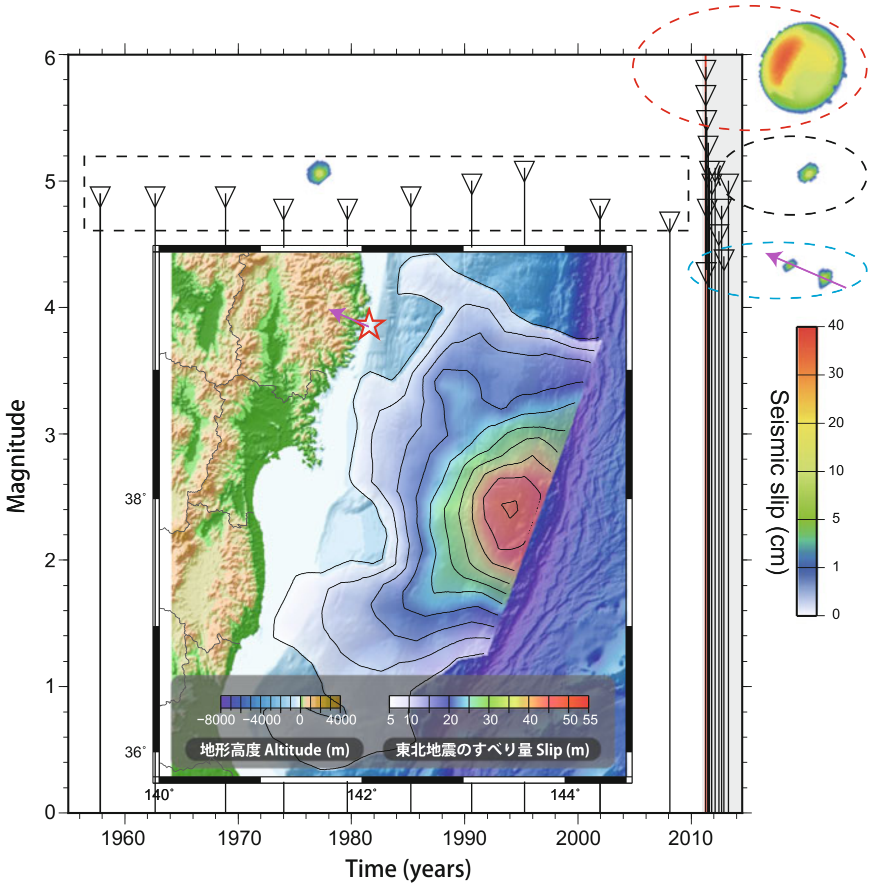

Fig. 2 Precursory Hypocenter (☆) and activity history of a repeating earthquake off Kamaishi. Borders between white and gray background colors areas represent the time at which the 2011 M9 Tohoku Earthquake occurred. Coseismic slip distributions off Kamaishi are shown with color scale on the right side. Arrows indicate the direction of postseismic slip estimated by numerical simulations on the basis of a rate- and state friction law (Ariyoshi et al. 2015)

source region. From numerical simulation as shown in Fig. 2, Ariyoshi et al. (2015) suggested that repeating earthquake occurs partial rupturing of the asperity without significant stress perturbation while complete rupturing and smaller doublet rupturing with significant stress perturbation due to the passage of large postseismic slip. In addition, Fig. 2 suggests that the stress release amount in the asperity is not homogeneous because of complex distribution of coseismic slip.

From recently statistical modeling, Nomura et al. (2017) applied the relation written as

$$\log T = -\log v + (1/6)\ \log M_o - 2.36 \tag{8}$$

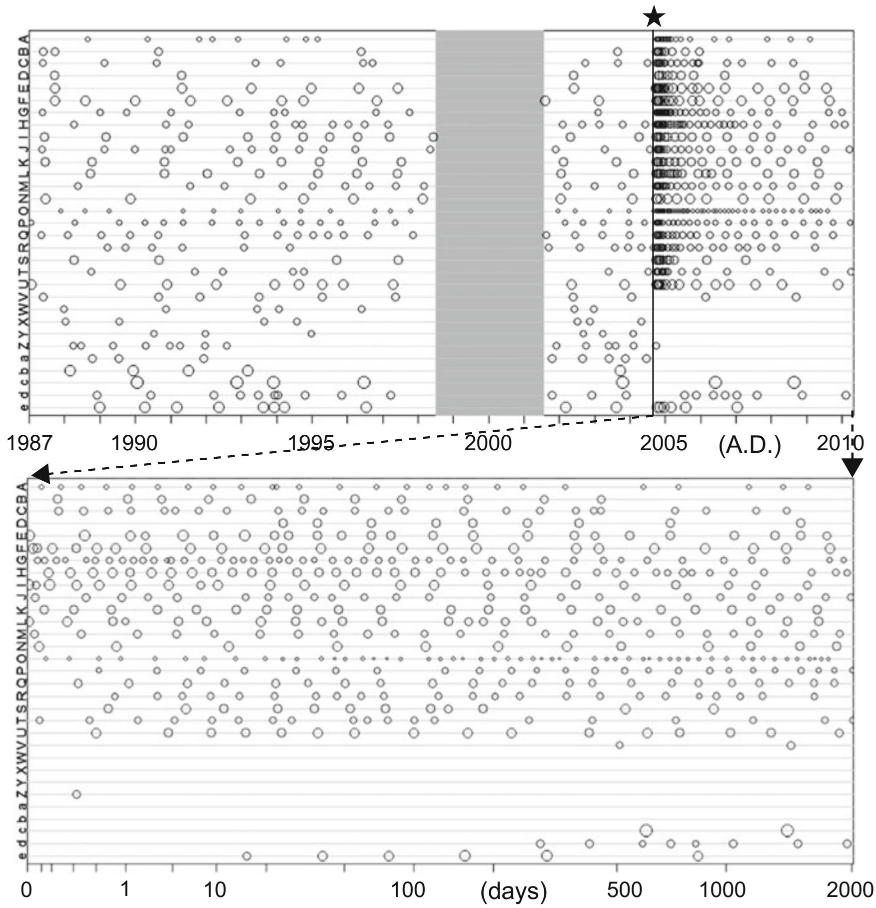

Fig. 3 The occurrence data of repeating earthquake sequences in Parkfield for each source region represented by indices (A–Z, a–e). The black star represents the occurrence time of mainshock (M6) on 28th September in 2004. The relative magnitude of each earthquake is reflected in the relative size of the symbols. The gray area represents unrecorded period. This figure is modified from Nomura et al. (2014)

to the non-stationary renewal model, in order to estimate slip velocity (v) by using the observed recurrence interval (T) and the seismic moment (M_o) as input parameters. This method has succeeded in estimating the spatiotemporal variations of interplate slip rate in northeast Japan. In order to forecast the occurrence time and the magnitude of the forthcoming earthquakes in advance, however, we have to take non-similar earthquakes such as doublet as shown in Fig. 2 and temporarily slow slip events (Ariyoshi et al. 2007) due to spatially inhomogeneous distribution of coseismic slip into account for moment release.

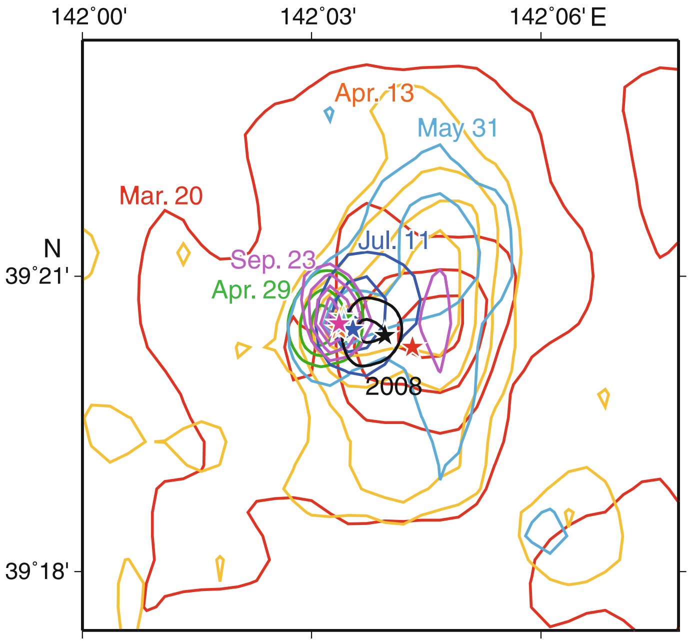

Fig. 4 Coseismic slip distribution (contour) and epicenter (star) just after the 2011 M9 Tohoku Earthquake. Black: 11 January 2008 earthquake, Red: 20 March 2011 for reference, Orange: 13 April 2011, Green: 29 April 2011, Light blue: 31 May 2011, Blue: 11 July 2011, Pink: 23 September 2011. The contours start at 5 cm and intervals are 20 cm for the 20 March 2011 event (red) and 5 cm for the other events. This figure is after Uchida et al. (2015)

From recent earthquake cycle modeling, Ariyoshi et al. (2007) showed that the simulated slip history of isolated small asperity with stress perturbation due to the passage of large postseismic slip far from the source region obeys the slip/time predictable model (Shimazaki and Nakata 1980) as shown in Fig. 5. Focusing on the stage just after the passage of the postseismic slip, Ariyoshi et al. (2015) pointed out that seismic moment release amount becomes temporally smaller due to triggered slow earthquakes and doublet rupturing along the propagation direction of postseismic slip.

4 Summaries and Future Plan

Our concept is summarized in Fig. 6. As our final goal, perfectly forecasting the next earthquake, is to identify its location, occurrence time and magnitude in advance. The location has been precisely determined on the basis of "asperity model" especially for repeating earthquakes as observed off Kamaishi (e.g., Igarashi et al. 2003). Without any stress perturbation, repeating earthquake is expected to be characteristic under the

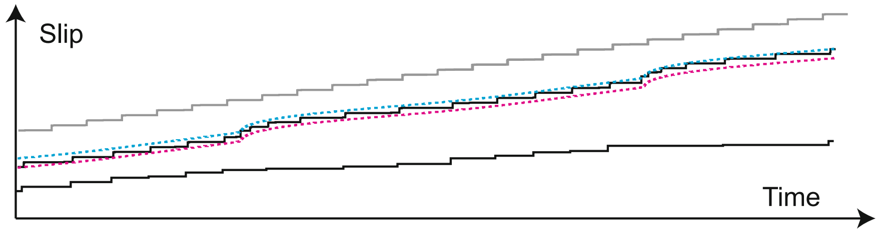

Fig. 5 Simulated time histories of total slip (black) sandwiched between the slip predictable model (cyan) and time predictable model (magenta) in case of the loading rate changed by postseismic slip. The top (gray) and bottom (black) curved represents coseismic slip of the small asperity in case of isolated asperity and two (small and large) asperities, respectively

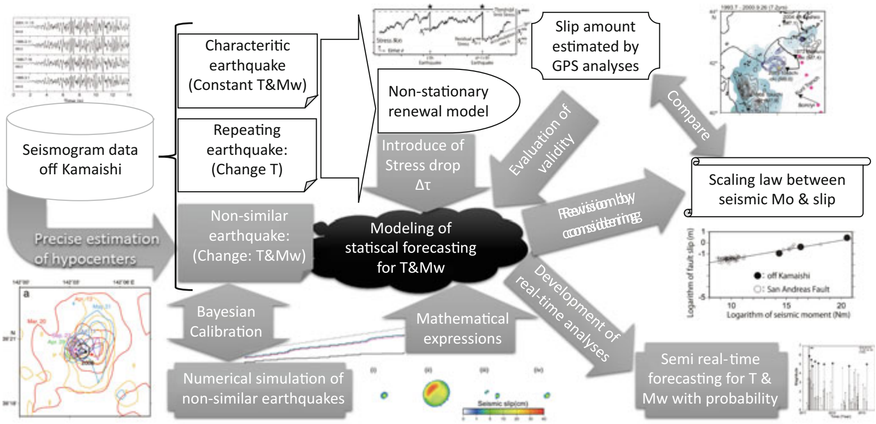

Fig. 6 Overview of seismological analysis for modeling of statistical forecasting occurrence time (T) and seismic moment magnitude (M_w). Gray background represents target study. Some parts of figure are after Igarashi et al. (2003), Uchida et al. (2009, 2015), Nomura et al. (2014) and Ariyoshi et al. (2007, 2015)

constant value of recurrence interval (T) and magnitude (M_w). With non-negligible stress perturbation, the recurrence time is also fluctuated under the condition that cross correlation is still high and approximately characteristic magnitude (±0.1). The estimation of the recurrence time has been improved by applying non-stationary renewal model (Nomura et al. 2015, 2017).

Just after the 2011 M9 Tohoku earthquake, however, magnitude of aftershocks occurred in the source area of repeating earthquakes tends to be fluctuated not only greater but also smaller with non-similarity in the observed seismograms due to great postseismic slip of megathrust earthquakes, which suggests that we should consider the effect of perturbed magnitude in order to enhance the forecast precision.

To overcome the difficult application to non-similar earthquake in case of the great postseismic slip, we have to understand the mechanism of the non-similar earthquake triggered by the passage of large postseismic slip from numerical simulations

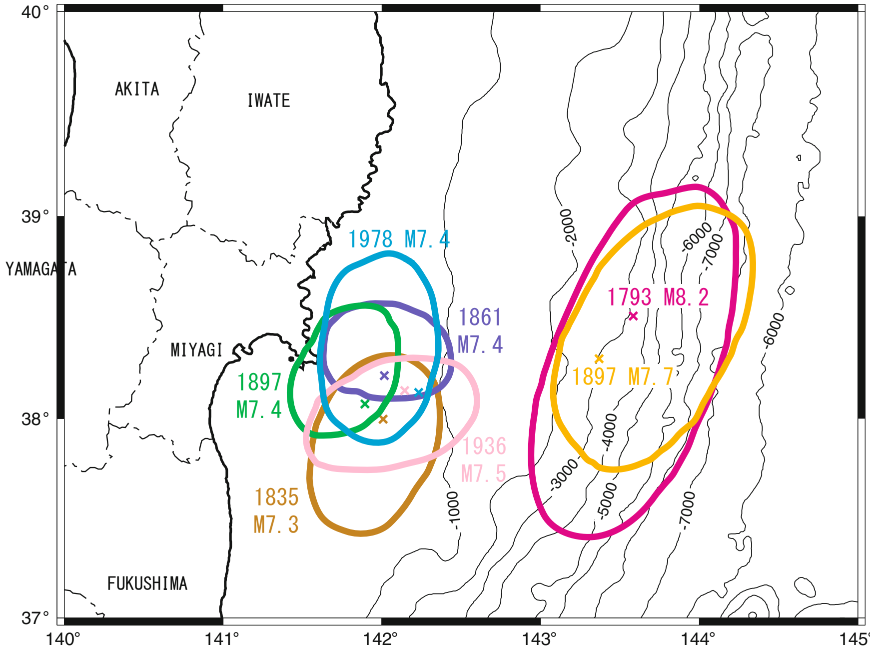

Fig. 7 Location of the source areas of Miyagi-oki (off Miyagi prefecture) earthquakes estimated by Hatori (1999). Colored stars represents the estimated epicenters

(Ariyoshi et al. 2015) based on rate- and state-dependent friction laws (Dieterich 1979; Ruina 1983) and dense network of seismological observations (Igarashi et al. 2010). Considering those simulated and observed results, we describe the temporal change of moment release amount as mathematical expressions toward the non-stationary renewal model.

Applying the scaling law between seismic moment and fault slip amount (Igarashi et al. 2003), we compare the estimated slip history between GPS data analysis and repeating earthquake (Uchida et al. 2009), which enables us to perform semi real-time analysis for evaluating the probability of occurrence time and magnitude of the next repeating earthquake as well as plate convergence rate in the future.

So far, most of repeating earthquakes have been observed as small to middle class ($Mw < 6$), while great interplate earthquakes such as off Miyagi prefecture (Fig. 7) and Nankai Trough (Fig. 8) have appeared to be approximately characteristic magnitude until the occurrence of M9 megathrust earthquakes.

Assuming that the difference of slip distribution for M7 earthquakes in Fig. 7 is caused by stress perturbation during longer period as interseismic stage, we can apply the generation mechanism of non-similar earthquake in the source region of repeating earthquake to the great earthquakes (M7 class) with perturbed source regions and recurrence intervals from a macroscopic perspective.

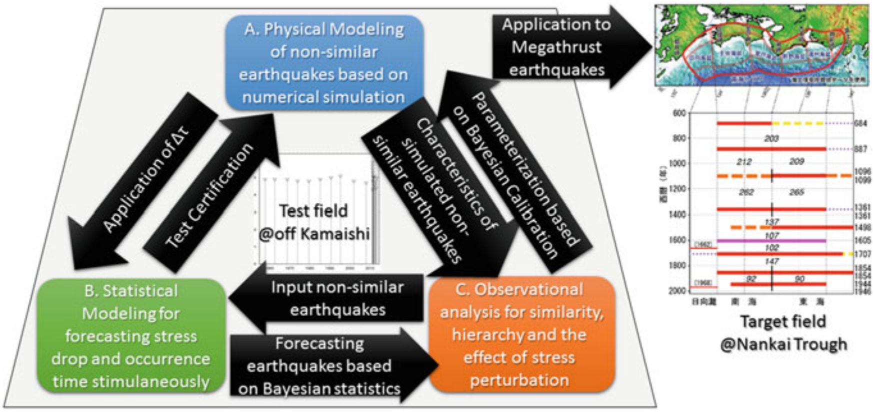

Fig. 8 Schematic diagram of our future study. Map view of Nankai Trough is from the Headquarters for Earthquake Research Promotion (2013)

Figure 8 shows schematic diagram of our future study. First, we adopt repeating earthquake off Kamaishi as test field, in order to combine (A) physical modeling of non-similar earthquakes by numerical simulation on the basis of rate- and state-dependent friction law (Ariyoshi et al. 2015), (B) statistical modeling for forecasting stress drop and occurrence time simultaneously on the basis of non-stationary renewal model, and (C) observational analysis for similarity and hierarchy (Uchida et al. 2015) with stress perturbation due to great postseismic slip. After confirmed in the test field, our modeling and understanding of non-similar earthquake would be applied to Nankai Trough as target field.

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